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Article

Pseudo Projective Tensor of Almost Kahler Manifold

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Abstract: The work from this research is to analyze the mathematical aspects of conharmonic tensor of projective Almost Kahler manifold. We get the Almost Kahler's manifold composites of the projecting tensor by using the cylindrical characteristics of the projected curvature tensor. And discovered several Kahler-like projective elements. Finally, demonstrated that the projections Almost Kahlar manifold's flat holomorphic sectional tensor is the manifold M. and shown that M is a Kahler manifold.

Keywords: Almost Kahler, Ricci Tensor, Pseudo Projective Tensor

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1. Introduction

The representation for one of the more recognized major groups of almost structure is $W_1 \oplus W_4$, where W_1 and $W_4$are respectively given by the conform local Kohler manifold. Grey & Harvalla [1] demonstrated that $W_1 \cup W_4 \subset W_1 \oplus W_4$. Many aspects of this subtype with a nearly Hermitian structure are strongly related to the features of nearly Kahler and locally condensed Kahler manifolds. After making initial efforts in 1960, Koto [2] discovered a relationship that was practically identical to the Kahler manifold and was regarded as a contribution to the manifold. We discovered the underpinning solution in the adjacent G-structure region [3]. Using the assumption that a group a part is authentic, Rakees [4] investigated the locally conformal Kahler of H_3 in 2008. Ali Shihab [5] investigated the formula within the conhormonic curved axis of a virtually Hermit manifold. Thus, the study went on until 1980, when Gray and Hervela published their most significant findings [6]. We have observed that the majority of research on this object has been conducted using the language of invariant Koszul [7]. The almost Hermitically manifold has been explored by several scholars [8].

2. Preliminaries

X(M) must have smooth in texture. M denotes the a vector space component. Let C^∞(M) denote a function group on M. The parallel space $T_P(M)$, $(J_P)^2 = id$ g=⟨.,.⟩ Matrix Riemannian on G [9]. The base is $[e_1 \dots, e_n \dots Je_1 \dots Je_n]$. The foundation $[i_1, \dots i_n, \dots \bar{i}_1, \dots \bar{i}_n]$. Is used to build the latest, this is denoted by { J, g }. With $i_a = \sigma(e_a)$ and $\bar{\sigma} = e_a$. The framework has been referred to as nearly structure. The producer's associated $[P, \dots i_1, \dots, i_n \dots, \bar{i}_1, \dots, \bar{i}_n]$ is an A-frame.

The measurements in the range of 1,…,2n are u, g, l, and P. The figures 1, 2,..., k will be used. With $\hat{a} = a + n$, use the indicators $[i_{\hat{1}} = \bar{i}_1, \dots i_{\hat{n}} = \bar{i}_n]$.You may write frame $[p, i_1 \dots, i_n \dots., i_{\hat{1}}, \dots i_{\hat{n}}]$ using that form. The parts of the matrix of the complicated structure with environment adjoined the following Q-structure kinds:

$$\left(\langle JX, JY\rangle J_j^i\right) = \begin{pmatrix} \sqrt{-1}\, I_n & 0 \\ 0 & -\sqrt{-1}\, I_n \end{pmatrix}, (g_j^i) = \begin{pmatrix} 0 & I_n \\ I_n & 0 \end{pmatrix} \qquad (1)$$

Which is the group system [10].

Definition 2.1 [11]

Assuming the measure g = <.,. > is a Riemann metrics while J is an essentially complex structure, the AH- A single set of vectors { J, g = < .,. > } constitute a framework on M, such that < JN, JM >=< N, M>.

Theorem 2.2 [12]

Almost Kahler-manifold structural variables are arranged in the attached G-structure as follows:

1) $d\phi^a = \phi_b^a \wedge \phi^b + B^{abc}\phi_b \wedge \phi_c;$
2) $d\phi_a = -\phi_a^b \wedge \phi_b B_{abc}\phi^b \wedge \phi^c;$
3) $d\phi_b^a \phi_c^a \wedge \phi_b^c + B_b^{adc}\phi_c \wedge \phi_d + B_{bcd}^a \phi^c \wedge \phi^d + \left(A_{bd}^{ac} + 2B^{ach}B_{hbd}\right)\phi^d \wedge \phi_c;$
4) $dB^{abc} = B_b^{abc}\phi^d + B^{abcd}\phi_d - B^{dbc}\phi_d^a + B^{adc}\phi_d^b + B^{adb}\phi_d^c;$

Definition 2.3 [14]

Consider it manifold, whose shape is provided by this connection. $a = \frac{1}{n-1}\mathcal{S}F o J$ is referred to as a Lie form, with S standing for the co-derivative. If is r-form, then whose opposite is a vector known as a Lie vector format while its co-derivative is (r-1)-form.

Theorem 2.4 [15]

The constituents of the Almost Kohler manifold's curved tensor in adjoin G-structure field take the subsequent types:

$1 - R_{bcd}^{\hat{a}} = 2B_{bcd}^a$ $\qquad$ $2 - R_{bcd}^a = -4B_{[c|ab|d]}$

$3 - R_{\hat{b}cd}^{\hat{a}} = -2B_{acd}^b$ $\qquad$ $4 - R_{\hat{b}\hat{c}\hat{d}}^a = -4B^{[c|ab|d]}$

$5 - R_{\hat{b}\hat{c}d}^a = -2B_d^{cab}$ $\qquad$ $6 - R_{b\hat{c}\hat{d}}^a = 2B_b^{adc}$

$7 - R_{\hat{b}c\hat{d}}^a = 2B^{dab}{}_c$ $\qquad$ $8 - R_{b\hat{c}d}^{\hat{a}} = 2B_{dab}{}^c$

$9 - R_{\hat{b}\hat{c}\hat{d}}^{\hat{a}} = B^{bcd}{}_a$ $\qquad$ $10 - R_{bc\hat{d}}^{\hat{a}} = 2B_{cab}{}^d$

$11 - R_{bc\hat{d}}^a = 2B^{adh}B_{hbc} - 4B^{dah}B_{cbh} + A_{bc}^{ad}$

$12 - R_{b\hat{c}d}^a = 4B^{cah}B_{dbh} - A_{bd}^{ac} - 2B^{ach}B_{hbd}$

$13 - R_{\hat{b}c\hat{d}}^{\hat{a}} = 4B^{dbh}B_{cah} - A_{ac}^{bd} - 2B^{bdh}B_{hac}$

$14 - R_{b\hat{c}\hat{d}}^{\hat{a}} = 4B^{hcd}B_{hab}$

$15 - R_{\hat{b}\hat{c}d}^{\hat{a}} = 2B^{bch}B_{had} + A_{ad}^{bc} - 4B_{dah}B^{cbh}$

$16 - R_{\hat{b}cd}^a = 4B^{hab}B_{hdc}$

Definition 2.5 [16]

The adjoin G-structure of a nearly manifold possesses N-invariant Ricci, if and only if $r_{ab} = r_b^{\hat{a}} = 0$.

3. Results and Discussion

Definition 3.1 [17]

The following is the definition of a pseudo projected scalar in a Riemann's manifold:

$$(P_{AK}) = \alpha R(X, Y, Z, W) + \beta[S(X, Z)g(Y, W) - r(Y, Z)g(X, W)] - \frac{K}{n}\left(\frac{\alpha}{n-1} + \beta\right)[g(X, Z)g(Y, W) - g(Y, Z)g(X, W)]$$

R is the Gaussian curving tensor, r is the Ricci tensor, and K is the linear curvature, with values α and β that do not equal zero.

Remark 3.2 [18]

The subsequent characteristics of the pseudo projects tensor are met:

1) $(P_{AK})(X, Y, Z, W) = -(P_{AK})(Y, X, Z, W)$
2) $(P_{AK})(X, Y, Z, W) = -(P_{AK})(Y, X, W, Z)$

where $\acute{X}, \acute{Y}, \acute{Z}, \hat{W} \in T_p(M)$.

Remark 3.3

If, $\bar{\alpha} = 1$ and $\beta = \frac{-1}{\eta-1}$, then the equation (1).

$$(P_{AK})_{ijkl} = (P)_{ijkl} = (R)_{ijkl} - \frac{1}{\eta - 1}[r_{ik}g_{jl} - r_{jk}g_{il}]$$

Theorem 3.4 [18]

Assuming M is a randomized AH-manifold of Almost Kohler is $B^{ab}_{c} = B^{c}_{ab} = 0$, $B^{\{abc\}} = B_{\{abc\}} = 0$.

Theorem 3.5

The adjacent G-structure contains the corresponding values for the elements of the Almost Kohler predictive tensor:

Proof:

1) Let $i = x, j = y, k = z$ and $l = w$

$$(P_{AK}) = \alpha R_{xyzw} + \beta[r_{xz}g_{yw} - r_{yz}g_{xw}] - \frac{K}{n}(\frac{\alpha}{n-1} + \beta)[g_{xz}g_{yw} - g_{yz}g_{xw}$$

By using theorem 2.5 we get

$$(P_{AK}) = 2\alpha B^{a}_{bcd}$$

2) Let $i = \hat{x}, j = y, k = z$ and $l = w$

$$(P_{AK}) = \alpha R_{\hat{x}yzw} + \beta[r_{\hat{x}z}g_{yw} - r_{yz}g_{\hat{x}w}] - \frac{K}{n}(\frac{\alpha}{n-1} + \beta)[g_{\hat{x}z}g_{yw} - g_{yz}g_{\hat{x}w}$$

By using theorem 2.5 we get

$$(P_{AK}) = -4\alpha B_{[c|ab|d]} - \beta r_{yz}\mathcal{S}^{x}_{w}$$

3) Let $i = x, j = \hat{y}, k = z$ and $l = w$

$$(P_{AK}) = \alpha R_{x\hat{y}zw} + \beta[r_{xz}g_{\hat{y}w} - r_{\hat{y}z}g_{xw}] - \frac{K}{n}(\frac{\alpha}{n-1} + \beta)[g_{xz}g_{\hat{y}w} - g_{\hat{y}z}g_{xw}$$

By using theorem 2.5 we get

$$(P_{AK}) = -2\alpha B^{b}_{acd} + \beta r_{xz}\mathcal{S}^{y}_{w}$$

4) Let $i = x, j = y, k = \hat{z}$ and $l = w$

$$(P_{AK}) = \alpha R_{xy\hat{z}w} + \beta[r_{x\hat{z}}g_{yw} - r_{y\hat{z}}g_{xw}] - \frac{K}{n}(\frac{\alpha}{n-1} + \beta)[g_{x\hat{z}}g_{yw} - g_{y\hat{z}}g_{xw}$$

By using theorem 2.5 we get

$$(P_{AK}) = 2\alpha B_{dab}{}^{c}$$

5) Let $i = x, j = y, k = z$ and $l = \hat{w}$

$$(P_{AK}) = \alpha R_{xyz\hat{w}} + \beta[r_{xz}g_{y\hat{w}} - r_{yz}g_{x\hat{w}}] - \frac{K}{n}(\frac{\alpha}{n-1} + \beta)[g_{xz}g_{y\hat{w}} - g_{yz}g_{x\hat{w}}$$

By using theorem 2.5 we get

$$(P_{AK}) = 2\alpha B\,{}^{d}_{cab} + \beta r_{xz}\mathcal{S}^{w}_{y} - \beta r_{yz}\mathcal{S}^{w}_{x}$$

6) Let $i = \hat{x}, j = \hat{y}, k = z$ and $l = w$

$$(P_{AK}) = \alpha R_{\hat{x}\hat{y}zw} + \beta[r_{\hat{x}z}g_{\hat{y}w} - r_{\hat{y}z}g_{\hat{x}w}] - \frac{K}{n}(\frac{\alpha}{n-1} + \beta)[g_{\hat{x}z}g_{\hat{y}w} - g_{\hat{y}z}g_{\hat{x}w}$$

By using theorem 2.5 we get

$(P_{AK}) = 4\alpha B^{hab}B_{hdc} + \beta r_{xz}\mathcal{S}_w^y - \beta r_{\hat{y}z}\mathcal{S}_w^x - \frac{K}{n}(\frac{\alpha}{n-1} + \beta)(\mathcal{S}_z^x\mathcal{S}_w^y - \mathcal{S}_z^y\mathcal{S}_w^x)$

7) Let $i = \hat{x}, j = y, k = \hat{z}$ and $l = w$

$(P_{AK}) = \alpha R_{\hat{x}y\hat{z}w} + \beta[r_{\hat{x}\hat{z}}g_{yw} - r_{y\hat{z}}g_{\hat{x}w}] - \frac{K}{n}(\frac{\alpha}{n-1} + \beta)[g_{\hat{x}\hat{z}}g_{yw} - g_{y\hat{z}}g_{\hat{x}w}$

By using theorem 2.5 we get

$$(P_{AK}) = 4\alpha B^{cah}B_{dbh} - \alpha A_{bd}^{ac} - 2\alpha B^{ach}B_{hbd} + \beta r_{y\hat{z}}\mathcal{S}_w^x + \frac{K}{n}(\frac{\alpha}{n-1} + \beta)(\mathcal{S}_y^z\mathcal{S}_w^x)$$

8) Let $i = x, j = \hat{y}, k = \hat{z}$ and $l = w$

$(P_{AK}) = \alpha R_{x\hat{y}\hat{z}w} + \beta[r_{x\hat{z}}g_{\hat{y}w} - r_{\hat{y}\hat{z}}g_{xw}] - \frac{K}{n}(\frac{\alpha}{n-1} + \beta)[g_{x\hat{z}}g_{\hat{y}w} - g_{\hat{y}\hat{z}}g_{xw}$

By using theorem 2.5 we get

$2\alpha B^{bch}B_{had} + \alpha A_{ad}^{bc} - 4\alpha B^{cbh}B_{adh} + \beta r_{x\hat{z}}\mathcal{S}_w^y - \frac{K}{n}(\frac{\alpha}{n-1} + \beta)(\mathcal{S}_x^z\mathcal{S}_w^y).$

Theorem 3.6

If the flat Rishi tensor is manifold of a flat pseudo projection tensor, then $\beta = \frac{-\alpha}{n-1}$

Proof:

Assuming M represents an AK manifold with a flat pseudo projective tensor, so $(P_{AK})_{abcd} = 0$

By using theorem 3.5 we get

$$4\alpha B^{cah}B_{dbh} - \alpha A_{bd}^{ac} - 2\alpha B^{ach}B_{hbd} + \beta r_{b\hat{c}}\mathcal{S}_d^a + \frac{K}{n}\left(\frac{\alpha}{n-1} + \beta\right)(\mathcal{S}_b^c\mathcal{S}_d^a) = 0$$

Given that M is the AK flat Rishi tensor, the following is true:

$$4\alpha B^{cah}B_{dbh} - \alpha A_{bd}^{ac} - 2\alpha B^{ach}B_{hbd} + \frac{K}{n}\left(\frac{\alpha}{n-1} + \beta\right)(\mathcal{S}_b^c\mathcal{S}_d^a) = 0$$

By index (b, d), we have symmetric and ant symmetric

$$\frac{K}{n}\left(\frac{\alpha}{n-1} + \beta\right)(\mathcal{S}_b^c\mathcal{S}_d^a) = 0$$

Thus $\beta = \frac{-\alpha}{n-1}$

Theorem 3.7

Suppose M be an AK manifold containing a pseudo projective tensor and a flat Rishi tensor; then M is a Kohler manifold.

Proof:

If M is an Almost kohler, then

using theorem 3.5 we obtain

$$4\alpha B^{hab}B_{hdc} + \beta r_{ac}\mathcal{S}_d^b - \beta r_{\hat{b}c}\mathcal{S}_d^a - \frac{K}{n}(\frac{\alpha}{n-1} + \beta)(\mathcal{S}_c^a\mathcal{S}_d^b - \mathcal{S}_c^b\mathcal{S}_d^a)$$

While AK is pseudo projects with a flat Rishi tensor, we get

$$4\alpha B^{hab}B_{hdc} - \frac{K}{n}\left(\frac{\alpha}{n-1} + \beta\right)\left(\mathcal{S}_c^a\mathcal{S}_d^b - \mathcal{S}_c^b\mathcal{S}_d^a\right) = 0$$

By theorem 3.6 we get

$$\beta = \frac{-\alpha}{n-1}, \text{ thus}$$

$$4\alpha B^{hab}B_{hdc} = 0 \Rightarrow B^{hab}B_{hdc} = 0$$

When we contraction the index (a, c), we get

$$B^{hab}B_{hab} = 0 \Rightarrow B^{hab}\overline{B}^{hab} = 0 \Rightarrow \sum_{h,a,b}\left(\left|B^{hab}\right|^2\right) = 0 \Leftrightarrow B^{hab} = 0$$

By theorem 3.4 we have

M is Kohler manifold.

Theorem 3.8

Suppose M is a holomorphic Metric if it is an AK-manifold with a pseudo projection tensor and a flat Rishi tensor.

Proof

If M is an AK manifold, then

By theorem 3.5 we get

$$-4\alpha B^{cbh}B_{adh}+2\alpha B^{bch}B_{had}+\alpha A^{bc}_{ad}+\beta r_{a\hat{c}}\mathcal{S}^{b}_{d}-\frac{K}{n}\Big(\frac{\alpha}{n-1}+\beta\Big)\left(\mathcal{S}^{c}_{a}\mathcal{S}^{b}_{d}\right)$$

While AK is pseudo projects with a flat Rishi tensor, we have

$$-4\alpha B_{dah}B^{cbh}+2\alpha B^{bch}B_{had}+\alpha A^{bc}_{ad}-\frac{K}{n}\Big(\frac{\alpha}{n-1}+\beta\Big)\left(\mathcal{S}^{c}_{a}\mathcal{S}^{b}_{d}\right)=0$$

By theorem 3.6 we get

$$-4\alpha B^{cbh}B_{adh}+2\alpha B^{bch}B_{had}+\alpha A^{bc}_{ad}=0$$

$$-4\alpha(\frac{1}{2}(ß^{cbh}ß_{adh}+ß^{bch}ß_{adh})+2\alpha(\frac{1}{2}(ß^{bch}ß_{had}+ß^{cbh}ß_{had})+\alpha A^{bc}_{ad}=0$$

$$-4\alpha(\frac{1}{2}(ß^{cbh}ß_{adh}-ß^{cbh}ß_{adh})+2\alpha(\frac{1}{2}(ß^{bch}ß_{had}-ß^{bch}ß_{had})+\alpha A^{bc}_{ad}=0$$

$$\alpha A^{bc}_{ad}=0\Rightarrow A^{bc}_{ad}=0$$

Therefore M is a holomorphic Sectional.

4. Conclusion

Investigations were conducted into the harmonic curvature of the Almost Kahler region, The experimental findings revealed the equivalence classes and their connections to the hierarchical manifold structure. Additionally, the standard curvature decline was computed for every unit, It denotes the curvature of the Remain that exemplifies certain models facilitating the examination of the typical structures within each category of the Hermitian manifold.

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