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Article

Methodology of Integrating Binomial Differential Expressions

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Abstract: This paper examines the problem of integrating binomial differential expressions, which are one of the important classes of indefinite integrals. In the study, the rational exponents appearing under the integral sign are identified, and the integration process is analyzed according to the cases where these exponents are integers, dividing the task into three main scenarios. For each case, methods for transforming the integral into an integral of rational functions using suitable substitutions are presented, and solutions are demonstrated through concrete examples related to the topic.

Keywords: Binomial, differential, rational exponent, integral, integral sign, constant.

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1. Introduction

Around Indefinite integrals play a fundamental role in higher mathematics, particularly in calculus and mathematical analysis, as they form the basis for solving differential equations, evaluating areas, and modeling various physical and engineering processes [1]. Among the wide variety of integrals encountered in practice, integrals involving binomial differential expressions occupy a special place due to their structural complexity and frequent appearance in both theoretical and applied problems [2].

The computation of integrals containing rational and irrational powers requires a solid understanding of substitution techniques and transformation methods. In particular, binomial differential expressions of the form

$$\int x^m (a + bx^n)^p dx$$

Where a and b are constant and m, n, and p are rational numbers, present notable challenges in integration. The difficulty of evaluating such integrals depends heavily on the relationships among the exponents involved [3].

A classical and systematic approach to addressing this problem was established through Chebyshev's theorem, which provides necessary and sufficient conditions under which binomial differentials can be reduced to integrals of rational functions. This theorem divides the problem into three principal cases based on the arithmetic properties of the rational exponents [4].

The aim of this paper is to analyze the integration of binomial differential expressions by applying Chebyshev's theorem. The study focuses on identifying the rational exponents involved and demonstrating how appropriate substitutions allow the reduction of the given integrals to rational forms. Concrete examples are provided to illustrate each case, thereby enhancing both theoretical understanding and practical computational skills [5].

2. Materials and Methods

The methodology of this study is based on analytical methods from integral calculus and algebraic transformation techniques. The research follows a structured approach consisting of the following stages:

1. Identification of the Binomial Differential Form

The general binomial differential expression under consideration is

$$\int x^m (a + bx^n)^p dx$$

where $a, b \in R$ and $m, n, p \in Q$. The first step involves determining the rational values of the exponents m, n, and p, as these values dictate the applicable integration strategy.

2. Application of Chebyshev's Theorem

According to Chebyshev's theorem, the given integral can be reduced to an integral of rational functions if at least one of the following conditions is satisfied:

p is an integer;

$\frac{m+1}{n}$ is an integer,

$\frac{m+1}{n} + p$ is an integer,

Each condition defines a separate integration case and determines the choice of substitution.

3. Reduction and Integration

After substitution, the integral is transformed into a rational function of the new variable. Standard techniques for integrating rational functions—such as polynomial integration and partial fractions—are then used to obtain the final result.

4. Verification Through Examples

Each case is illustrated with explicit examples to verify the correctness and effectiveness of the applied methods and to demonstrate their practical implementation.

3. Results

Knowledge of the methods for calculating indefinite integrals and their fundamental formulas is of significant importance in integrating binomial differential expressions [6]. In particular, a sufficient understanding and skill in calculating integrals of rational and irrational functions is required. On this basis, the present study examines the integration of binomial differential expressions of the following form, problems that allow convenient computation [7]:

$$x^m (a + bx^n)^p dx \quad (1)$$

we study the problem of integrating a differential expression, that is, its amenability to convenient computation [8].

Definition: The following expression

$$x^m (a + bx^n)^p dx$$

is called a **binomial differential expression**, where a, b are constants, and m, n, p are rational numbers.

Now, consider the integral of binomial differentials:

$$\int x^m (a + x^n)^p dx \quad (2)$$

Clearly, the evaluation of this integral depends on the values of the rational numbers m, n, p. The famous Russian mathematician P.L. Chebyshev proved that the integral (1) can be expressed in terms of the integral of rational functions in one of the following three cases [9]:

1) p is an integer
2) $\frac{m+1}{n}$ is an integer

3) $\frac{m+1}{n}+p$ is an integer.

Thus, let us consider the first case where p is an integer. In this case, m and n are chosen σ the **least common multiple** of the denominators of the rational numbers, and with the substitution x=t integral (2), the integrand becomes a rational function, so integral (2) reduces to the integral of a rational function [10].

Example: Consider the integra

$$\int \frac{\sqrt{x}}{(1+\sqrt[3]{x})}dx \qquad (3)$$

Comparing this integral with integral (1),

$$\int \frac{\sqrt{x}}{(1+\sqrt[3]{x})}dx = \int \sqrt{x}(1+\sqrt[3]{x})^{-2}dx = \int x^{\frac{1}{2}}(1+x^{\frac{1}{3}})^{-2}dx$$

we find that $p=-2$. When p is an integer as above, after performing the substitution, we can compute this integral. Since $m=\frac{1}{2}, n=\frac{1}{3}$ EKUK(3,2)=6, taking this into account, the integral becomes

$$x = t^6$$

$$\int x^{\frac{1}{2}}(1+x^{\frac{1}{3}})^{-2}dx = 6\int \frac{t^3}{(1+t^2)^2}dt =$$

$$= 6\int (t^4 - 2t^2 + 3 - 4\cdot\frac{1}{t^2+1} + \frac{1}{(t^2+1)^2})dt =$$

$$= \frac{5}{6}t^5 - 4t^3 + 18t - \frac{2}{arctgt} + 3\cdot\frac{t}{t^2+1} + c$$

Taking into account that $t=\sqrt[6]{x}$, our integral becomes:

$$\int \dots dx$$

$$\int \frac{\sqrt{x}}{(1+\sqrt[3]{x})}dx = \frac{6}{5}\sqrt[6]{x^5} - 4\sqrt{x} + 18\sqrt[6]{x} - \frac{2}{arctg\sqrt[6]{x}} + \frac{3\sqrt[6]{x}}{\sqrt[3]{x}+1} + c.$$

2. If $\frac{m+1}{n}$ is an integer, then with an appropriate substitution $z=(a+bx^n)^{\frac{1}{s}}$ in integral (2), it reduces to the integral of a rational function.

3. If p+q is an integer, then after substitution $z=(\frac{a+bx^n}{x^n})^{\frac{1}{s}}$, integral (2) reduces to the integral of a rational function.

Example: Consider the integral

$$\int \frac{x^5}{\sqrt{1-x^2}}dx$$

Solution: First, find p and m, n. Here p=$-\frac{1}{2}$, m=5 , n=2 . Since p is not an integer, we check condition 2): $\frac{m+1}{n}=\frac{5+1}{2}=3$ an integer. Hence, we perform the substitution $z=\sqrt{1-x^2}$.

$$\int \frac{x^5}{\sqrt{1-x^2}}dx = \begin{bmatrix} t = \sqrt{1-x^2} \\ dz = -\frac{2x}{2\sqrt{1-x^2}}dx \\ x^2 = 1-t^2 \end{bmatrix} =$$

$$= \int \frac{(1-z^2)^2 \cdot x}{z} \cdot \frac{\sqrt{1-x^2}}{-x}dz = -\int (1-z^2)^2 dz = -z + \frac{2}{3}z^2 - \frac{1}{5}z^5 + c$$

Taking into account that $z = \sqrt{1-x^2}$, we obtain the expression

$$\int \frac{x^5}{\sqrt{1-x^2}}dx = -\sqrt{1-x^2} + \frac{2}{3}(1-x^2) - \frac{1}{5}(1-x^2)^2 \cdot \sqrt{1-x^2} + c$$

The findings from the study also indicate that R theory is effective and systematic in unifying binomial differences. When classified by the arithmetical nature of their rational exponents, integrals become a little more systematic and foreseeable [11].

The examples elaborated in this paper illustrate that when an integral initially seems to be ugly and complicated because of irrational powers, then the right substitution can make it pretty nice. More importantly, binomial reduction of differentials to rational integrals means the application of powerful methods from classical integration is straightforward and rapidly implemented [12]. From an educational standpoint, it has been observed that high school students who wish to study more advanced mathematics find this type of knowledge particularly important [13]. It serves as a bridge between the theoretical underpinnings of machine learning and applicable problem solving techniques. Additionally, it offers a taste of the techniques employed in more advanced work like differential equations, mathematical physics and applied analysis [14].

In general, the approach that we have discussed in this paper not only reduces the effort to handle a large class of integrals but also emphasizes on the role of algebraic wisdom in calculus. Chebyshev's conditions being so systematic, some definite statements about binomial differential integrals can be made with confidence and clearness [15].

4. Conclusion

In this paper, the main methodology for integrating binomial differential expressions was presented based on Chebyshev's theorem. Depending on the values of the rational exponents, the integral was systematically divided into three main cases, and the corresponding methods for calculation were examined. For each case, it was demonstrated that, using appropriate substitutions, the integral can be reduced to an integral of rational functions. The examples provided serve to practically illustrate the methods for integrating binomial differentials. This methodology is of significant importance for students in higher mathematics courses, as it helps in a deeper understanding of the topic of indefinite integrals.

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